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Relative Dispersion and Achromatism.

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(From the Research Laboratory, Messrs. Chance Brothers & Co., Ltd.)

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THE term Relative Dispersion may be said to refer to the ratio between the dispersion of a medium for any pair of spectral lines to that for some other pair. In its more definite and restricted meaning the given dispersion is compared with a standard dispersion, almost universally taken as that between the lines C(H_α) and F(H_β) of the low pressure hydrogen spectrum. For the purpose of this paper we shall refer to the relative dispersion for a given line X as meaning the ratio of the dispersion between C and that line to the standard dispersion C – F, so that

$$\text{Relative Dispersion} = \frac{n_X - n_C}{n_F - n_C}$$

Fraunhofer first realised that the values of such ratios for different glasses were not identical. The lack of exact proportionality in the dispersions (irrationality of dispersion) makes it impossible to achromatise a crown and flint lens for more than two colours, say C and F, and the yellow and green rays focus slightly inside, deep red and blue violet rays slightly outside the focus for C and F, resulting in slight outstanding fringes of apple green or claret colour. This colour effect is spoken of as "secondary spectrum." The want of proportionality in dispersion produced by borosilicate crown and dense flint is represented to scale

in Fig. 1 by the differences in the spectra produced by two prisms of these glasses adjusted to give equal dispersions for C and F.

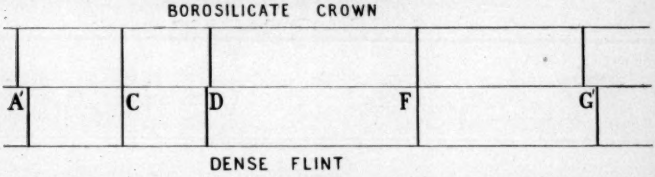


Fig. 1.

The effects of secondary spectrum are not sufficiently noticeable to be of importance in photographic lenses, and in objectives of small size or large relative aperture, in which spherical and other aberrations are present to a greater extent. In objectives of higher precision in axial corrections, and particularly in great astronomical objectives, secondary spectrum is a more serious defect. Thus, in the case of the objective of the Lick Telescope of focal length 17·4 metres, achromatised for B and F, the focus for G^1 is 36·8 mm. and for *h* 70·1 mm. outside that for the achromatised rays.

Fraunhofer was acquainted with the fact that the dispersion of flint glass was almost exactly matched by oil of turpentine, and hoped that by changes of composition in glasses irrationality of dispersion could be diminished or eliminated. Working at the manufacture of optical glass he did, in fact, achieve something in this direction, but the glasses in question were not of sufficient durability.

Harcourt and later Stokes carried out more elaborate researches beginning in 1834, using 20 fresh elements in the composition of their glasses. They discovered the effect of boric and phosphoric acids in reducing dispersion, but the prisms of their glasses were not sufficiently homogeneous to allow accurate measurement of relative dispersions, though they proved the possibility of reducing secondary spectrum.

The researches of Schott and Abbe resulted in the production on a large scale of glasses which reduced and in one case practically abolished secondary spectrum (according to Jena figures). The most efficient glasses for the purpose—phosphate crowns and borate flints—proved, however, to be insufficiently durable and were withdrawn. Among glasses of a fairly good durability there remain only "telescope" flints of rather low dispersion to match with crowns not widely different in dispersion.

On taking charge of research work at Messrs. Chance Brothers, one of us gave early attention to this subject, and plotted the relative dispersion

for G^1 against ν for the glasses in the Jena list, and so obtained points which were found to lie roughly on a line. It was determined to investigate this question in a more searching manner with a spectrometer of high precision then on order for us. A few months later Prof. Conrady showed one of us a similar diagram in which the three relative dispersions given in the Jena list were plotted against ν giving points lying fairly closely on three lines, and doubt was expressed whether anything had been achieved by Schott in approximating the relative dispersions of "telescope" flints and crowns except in proportion to sacrifice of difference in dispersive power.

Shortly afterwards Col. Gifford, who had kindly consented to measure for us reference prisms for the standardisation of our Pulfrich refractometer, put before us a summary of his measurements of indices with new corrections for barometric pressure. We then repeated the graphical representation of ν and relative dispersion and obtained points which did not lie so closely on a line as those obtained from the Jena list; in fact there seemed to be some indication that the run of dispersion was more closely related to the mean dispersion $n_F - n_C$.

If glasses of the same ν value exhibit a similar run of dispersion there can be no possibility of reducing secondary spectrum except by sacrificing to a proportionate extent the difference in the dispersive power of two glasses. This sacrifice involves deeper curves in an achromatic doublet, increased spherical aberration and colour differences of spherical aberration, and thicker glass, the last named alone being a very important defect in the largest objectives from the point of view of production of the glass and absorption of light. If, on the other hand, glasses of the same mean dispersion have a similar run of dispersion, a dense barium crown and light flint of equal dispersion and different ν could be chosen to give complete achromatism; moreover, strict interpolation of indices between two glasses would be possible and it would be unnecessary ever to measure more than two indices for a glass.

It therefore appeared worth while investigating the data as fully as possible to ascertain to what extent either of the above propositions was true. For this purpose we undertook to apply a smoothing process to the observations of Col. Gifford to correct as far as possible outstanding sources of error, and we considered that the large number of glasses, and of indices for each glass measured, gave us a means of doing this. It would be impossible to carry this out by representing graphically the actual indices in terms, say, of the wave lengths of the lines, as one is seeking to correct errors in the indices which are minute in comparison with the indices themselves. The various relative dispersions are on the other hand much more nearly the same for all glasses, so that if we consider the departures of the relative dispersions from the mean value

TABLE I.

Relative Dispersions.

	Observed.			K 7682	He 7066	Li 6708	H 6563	Na 5893	Pb 5607	Hg 5461	Fe 5270	H 4861	Cd 4678	Cd 4415	H 4341	Hg 4046	Description.
	n_D	$n_F - n_C$	ν	A^1	B^1	Li	C	D	A	Hg_1	E	F	Φ	Cd_7	G^1	Hg_2	
Number 28	1·4881	·00691	70·6	—·3649	—·1832	—·0565	0	·3019	·4591	·5481	·6785	1	1·1730	1·4598	1·5525	1·9688	Fluor Crown
1	1·4996	·00758	65·9	—·3608	—·1788	—·0548	0	·3008	·4558	·5488	·6755	1	1·1755	1·4618	1·5508	1·9724	Borosilicate Crown
35	1·5088	·00794	64·1	—·3543	—·1768	—·0554	0	·2977	·4559	·5456	·6754	1	1·1779	1·4673	1·5593	1·9803	,, ,,
38	1·5089	·00802	63·5	—·3541	—·1772	—·0555	0	·2980	·4559	·5457	·6765	1	1·1779	1·4701	1·5631	1·9832	,, ,,
3	1·5123	·00804	63·7	—·3555	—·1759	—·0541	0	·2977	·4531	·5444	·6725	1	1·1759	1·4674	1·5618	1·9851	,, ,,
4	1·5159	·00806	64·0	—·3610	—·1811	—·0552	0	·2978	·4547	·5484	·6780	1	1·1737	1·4671	1·5527	1·9727	,, ,,
5	1·5100	·00807	63·2	—·3501	—·1722	—·0514	0	·2980	·4554	·5452	·6723	1	1·1723	1·4641	1·5571	1·9759	,, ,,
6	1·5015	·00826	60·7	—·3465	—·1714	—·0497	0	·2980	·4519	·5451	·6729	1	1·1726	1·4694	1·5639	1·9891	,, ,,
29	1.5291	·00851	62·2	—·3500	—·1720	—·0540	0	·2977	·4545	·5461	·6741	1	1·1785	1·4686	1·5596	1·9883	,, ,,
8	1·5165	·00853	60·6	—·3576	—·1770	—·0592	0	·2954	·4513	·5428	·6752	1	1·1770	1·4689	1·5627	1·9923	Silicate Crown
31	1·5271	·00875	60·2	—·3469	—·1760	—·0549	0	·2983	·4532	·5429	·6737	1	1·1812	1·4789	1·5703	2·0006	,, ,,
9	1·5308	·00877	60·5	—·3440	—·1734	—·0536	0	·2966	·4518	·5442	·6708	1	1·1791	1·4775	1·5716	2·0017	Telescope Crown
10	1·5121	·00890	57·5	—·3455	—·1775	—·0562	0	·2978	·4506	·5449	·6714	1	1·1809	1·4764	1·5736	2·0045	Zinco Silicate Crown
32	1·5407	·00909	59·5	—·3489	—·1734	—·0528	0	·2939	·4540	·5437	·6698	1	1·1761	1·4711	1·5630	1·9950	Barium Silicate Crown
11	1·5714	·00995	57·4	—·3437	—·1709	—·0523	0	·2935	·4513	·5422	·6709	1	1·1764	1·4754	1·5688	2·0030	Densest Barium SilicateCrown
12	1·5680	·01023	55·5	—·3462	—·1736	—·0538	0	·2914	·4494	·5369	·6704	1	1·1800	1·4822	1·5756	2·0176	Baryta Light Flint
13	1·5490	·01026	53·5	—·3387	—·1711	—·0512	0	·2934	·4498	·5414	·6696	1	1·1818	1·4815	1·5789	2·0214	,, ,, ,,
14	1·5294	·01028	51·5	—·3474	—·1737	—.0564	0	·2949	·4516	·5411	·6691	1	1·1786	1·4774	1·5698	2·0127	Telescope Flint
15	1·6098	·01030	59·2	—·3557	—·1752	—.0607	0	·2921	·4493	·5410	·6701	1	1·1766	1·4731	1·5687	1·9985	Dense Barium Crown
16	1·6087	·01034	58·9	—·3475	—·1745	—·0537	0	·2924	·4500	·5404	·6723	1	1·1803	1·4727	1·5665	2·0005	Heavy Barium Crown
17	1·5314	·01037	51·2	—·3420	—·1741	—·0531	0	·2928	·4477	·5388	·6710	1	1·1761	1·4785	1·5750	2·0145	Telescope Flint
18	1·6107	·01096	55·7	—·3412	—·1706	—·0552	0	·2920	·4498	·5397	·6679	1	1·1784	1·4754	1·5721	2·0078	Dense Baryta Crown
22	1·5831	·01252	40·6	—·3351	—·1693	—·0523	·0	·2887	·4429	·5331	·6641	1	1·1833	1·4908	1·5891	2·0471	Baryta Light Flint
24	1·5694	·01320	43·1	—·3259	—·1629	—·0485	0	·2910	·4460	·5335	·6643	1	1·1845	1·4937	1·5938	2·0587	Light Flint
25	1·5794	·01374	42·2	—·3282	—·1663	—·0517	0	·2864	·4414	·5327	·6648	1	1·1812	1·4931	1·5939	2·0597	,, ,,
26	1·6210	·01568	39·6	—·3221	—·1633	—·0510	0	·2867	·4407	·5328	·6617	1	1·1846	1·4981	1·5992	2·0743	Baryta Flint
27	1·6121	·01656	37·0	—·3227	—·1642	—·0510	0	·2849	·4404	·5294	·6610	1	1·1859	1·5038	1·6076	2·0863	Dense Flint
33	1·6130	·01658	37·0	—·3242	—·1647	—·0513	0	·2856	·4412	·5302	·6616	1	1·1867	1·5048	1·6077	2·0884	,, ,,
36	1·6190	·01698	36·5	—·3219	—·1626	—·0521	0	·2857	·4395	·5314	·6630	1	1·1859	1·5022	1·6053	2·0836	,, ,,
37	1·6197	·01709	36·3	—·3217	—·1628	—·0527	0	·2855	·4398	·5318	·6620	1	1·1869	1·5059	1·6097	2·0909	,, ,,
Average				—·3435	—·1722	—·0537	0	·2936	·4496	·5404	·6700	1	1·1793	1·4792	1·5749	2·0158	

At the top of the Table, the element, wave-length and title are given for each line.

TABLE II.

Differences between Relative Dispersions and Standard Relative Dispersions.

Number	A^1	B^1	Li	C	D	A	Hg_1	E	F	Φ	Cd_7	G^1	Hg_2
28	− 214	− 110	− 28	0	+ 83	+ 95	+ 77	+ 85	0	− 63	− 194	− 224	− 470
1	− 173	− 66	− 11	0	+ 72	+ 62	+ 84	+ 55	0	− 38	− 174	− 241	− 434
35	− 108	− 46	− 17	0	+ 41	+ 63	+ 52	+ 54	0	− 14	− 119	− 156	− 355
38	− 106	− 50	− 18	0	+ 44	+ 63	+ 53	+ 65	0	− 14	− 91	− 118	− 326
3	− 120	− 37	− 4	0	+ 41	+ 35	+ 40	+ 25	0	− 34	− 118	− 131	− 307
4	− 175	− 89	− 15	0	+ 42	+ 51	+ 80	+ 80	0	− 56	− 121	− 222	− 431
5	− 66	0	+ 23	0	+ 44	+ 58	+ 48	+ 23	0	− 70	− 151	− 178	− 399
6	− 30	+ 8	+ 40	0	+ 44	+ 23	+ 47	+ 29	0	− 67	− 98	− 110	− 267
29	− 65	+ 2	− 3	0	+ 41	+ 49	+ 57	+ 41	0	− 8	− 106	− 153	− 275
8	− 141	− 48	− 55	0	+ 18	+ 17	+ 24	+ 52	0	− 23	− 103	− 122	− 235
31	− 34	− 38	− 12	0	+ 47	+ 36	+ 25	+ 37	0	+ 19	− 3	− 46	− 152
9	− 5	− 12	+ 1	0	+ 30	+ 22	+ 38	+ 8	0	− 2	− 17	− 33	− 141
10	− 20	− 53	− 25	0	+ 42	+ 10	+ 45	+ 14	0	+ 16	− 28	− 13	− 113
32	− 54	− 12	+ 9	0	+ 3	+ 44	+ 33	− 2	0	− 32	− 81	− 119	− 208
11	− 2	+ 13	+ 14	0	− 1	+ 17	+ 18	+ 9	0	− 29	− 38	− 61	− 128
12	− 27	− 14	− 1	0	− 22	− 2	− 35	+ 4	0	+ 7	+ 30	+ 7	+ 18
13	+ 48	+ 11	+ 25	0	− 2	+ 2	+ 10	− 4	0	+ 25	+ 23	+ 40	+ 56
14	− 39	− 15	− 27	0	+ 13	+ 20	+ 7	− 9	0	− 7	− 18	− 51	− 31
15	− 122	− 30	− 70	0	− 15	− 3	+ 6	+ 1	0	− 27	− 61	− 62	− 73
16	− 40	− 23	0	0	− 12	+ 4	0	+ 23	0	+ 10	− 65	− 84	− 153
17	+ 15	− 19	+ 6	0	− 8	− 19	− 16	+ 10	0	− 32	− 7	+ 1	− 13
18	+ 23	+ 16	− 15	0	− 16	+ 2	− 7	− 21	0	− 9	− 38	− 28	− 80
22	+ 84	+ 29	+ 14	0	− 49	− 67	− 73	− 59	0	+ 40	+ 116	+ 142	+ 313
24	+ 176	+ 93	+ 52	0	− 26	− 36	− 69	− 57	0	+ 52	+ 145	+ 189	+ 429
25	+ 153	+ 59	+ 20	0	− 72	− 82	− 77	− 52	0	+ 19	+ 139	+ 190	+ 439
26	+ 214	+ 89	+ 27	0	− 69	− 89	− 76	− 83	0	+ 53	+ 189	+ 243	+ 585
27	+ 208	+ 80	+ 27	0	− 87	− 92	− 110	− 90	0	+ 66	+ 246	+ 327	+ 705
33	+ 193	+ 75	+ 24	0	− 80	− 84	− 102	− 84	0	+ 74	+ 256	+ 328	+ 726
36	+ 216	+ 96	+ 16	0	− 79	− 101	− 90	− 70	0	+ 66	+ 230	+ 304	+ 678
37	+ 218	+ 94	+ 10	0	− 81	− 98	− 86	− 80	0	+ 76	+ 267	+ 348	+ 751

The unit is 10^{-4} throughout.

for a large number of glasses, we are then dealing with quantities comparable in size with the variations due to experimental error.

Col. Gifford had given us measurements of indices for 30 glasses, each for 13 lines in the spectrum. These indices were given to five decimal places and "raised" values were indicated, *i.e.*, the numbers were significant to ·000005.

The indices were all obtained by absolute determinations with equilateral prisms, as described by Col. Gifford in Proc. Roy. Soc. LXX, p. 329, 1902, and in each case refraction coefficients with temperature, and barometric corrections were used to ensure accuracy. We therefore calculated the relative dispersions as defined above for all the lines in each of these glasses, and took the mean of the results obtained. In Table I. are given the values of the relative dispersions for the different glasses, arranged in ascending order of mean dispersion, and also the means of these values. The type of glass in each case is shown in the last column of the Table.

The mean values for the relative dispersions we shall for our purpose speak of as "standard relative dispersions," but the actual numbers depend wholly of course on the numbers and kinds of glasses dealt with.

The departures of the relative dispersion for each glass from the standard relative dispersions were then calculated, the results being given in Table II. It will be seen (bearing in mind that the relative dispersions for lines less refrangible than C are negative) that this table exhibits the extents to which the different regions of the spectrum are compressed or extended with the various glasses, as indicated graphically in Fig. 1. The values in Table II. were then taken as ordinates and plotted in terms of the standard relative dispersions as abscissæ. Glasses of low dispersion gave curves of the type indicated in Fig. 2 by the fluor crown curve, and glasses of high dispersion a similar type of curve inverted. Two such actual curves (before correction for C and F) for the extreme glasses of the series—fluor crown 28 and dense flint 37—are shown in Fig. 2.

The meaning of these curves will be realised by reference to Fig. 1. If for a moment we regard the B.S.C. glass in this figure as one which has standard values of relative dispersions, when the spectrum given by this glass is compared with one given by a dense flint prism adjusted to produce equal dispersion for C – F, it will be noted that the lines A^1 and G^1 are shifted in one direction and D in the other (negative) direction by amounts represented by the ordinates of the points on the curves. The ordinates also represent (approximately) the variations in the focal length for different lines of a doublet composed of a glass of "standard" relative dispersions achromatised for C and F with the glass in question.

Smooth curves were then drawn among the observed points so obtained in order to make evident errors of observation, and the values for any selected relative dispersion corrected to the curve were then taken as being more accurate values for our investigations.

In general the smooth curves did not pass accurately through the axis at C and F, there being experimental error for these as well as for other lines. Any such error causes an error in the values of all the relative dispersions which are calculated in terms of the interval C − F, and the error in the measurements for C and F must be taken into account in correcting the relative dispersions of all the other lines. After these corrections had been applied it was found that the three glasses 12, 17, 18, gave relative dispersions the graphical representation of which showed

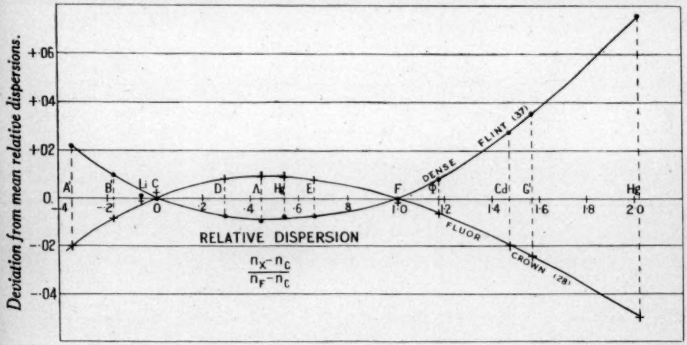


Fig. 2.

them as nearly as could be detected to be coincident (apart from experimental error) with the standard relative dispersions. That is, the run of dispersion which is the average of the set of glasses considered is exhibited throughout the spectrum as closely as may be ascertained from the data by three actual glasses.

More accurate values of the relative dispersion for D and G^1 were calculated from the smoothed curves, and these were represented graphically in terms of the mean dispersion and the reciprocal dispersive power (ν), the results being given in Figs. 3 to 5.*

* Some of the above diagrams were not exhibited when this paper was read, and a few of the above conclusions were not set forth. This should be borne in mind in reading the discussion.

From the diagrams it will be seen that the view that the relative dispersions are functions of the dispersive power is not strictly true within the limits of experimental error of the reduced observations, nor on the other hand are the relative dispersions accurate functions of the mean dispersions. Relative dispersion for the D line seems rather more closely a function of the mean dispersion than of the dispersive power, but the reverse is the case for the G^1 line. That the above statement is not inherently impossible or improbable is evident from the consideration that the forms of the curves of relative dispersion differences are often not exactly similar for different glasses, and in a few cases differ markedly.

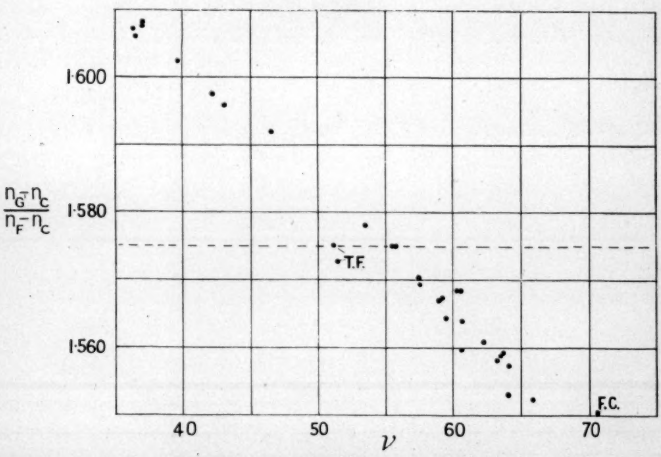


Fig. 3.

The dotted line indicates the mean relative dispersion for G^1.

On looking into the results obtained it is evident that in the " telescope " flints produced first at Jena—and now reproduced by Mantois and Chance —something has been definitely achieved in the direction of reduction of secondary spectrum by change of composition apart from approximation of dispersive powers, more in fact than would appear from the figures for the relative dispersions for G^1 in the Jena catalogue. Unfortunately this result has only been obtained at the expense of some reduction in the essential quality of durability. As mentioned above, three glasses,

12, 17, and 18 showed an identical run of dispersion to within the limits of experimental error, and by using telescope flint 17 with dense barium crown 18 or light barium flint 12 very good achromatism should result over the whole visible spectrum, but very deep curves would be required on account of the small difference in ν.

Other statements in Hovestadt's "Jena Glass" are also substantiated. Thus it will be noticed that the relative dispersion for G^1 of the fluor

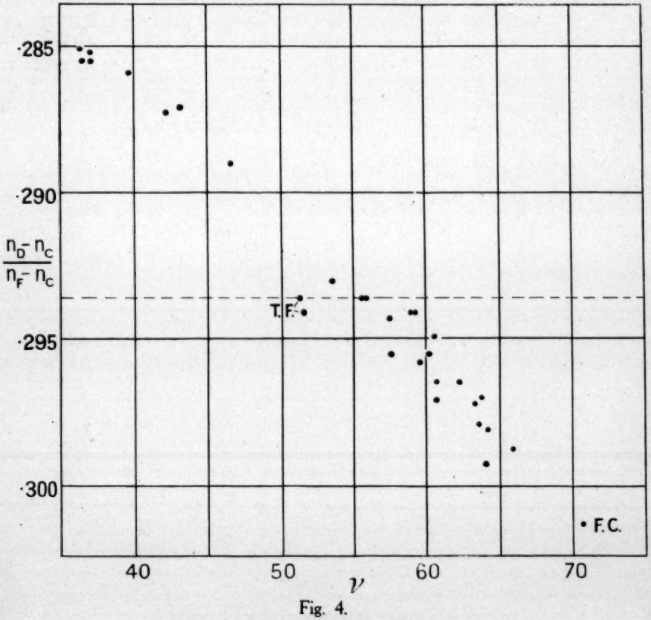


Fig. 4.

Mean relative dispersion indicated by dotted line.

crown 28 is higher than would be expected from the value of ν, *i.e.*, it lengthens out the violet end and would be favourable for matching with a flint. The red end, however, is not proportionately compressed (see Fig. 2), and hence, whilst the point representing the fluor crown on Fig. 3 for G^1 departs from the general trend of the other glasses it will

be found to be well in line on a similar diagram for A^1. Reverse considerations to a much less extent are found to hold good in the case of boro-silicate crown, but the indications are not so certain as might be wished.

If we consider two glasses for which the curves differ as they would for fluor crown and boro-silicate crown, but to a greater extent, it will be seen that the fact of one curve being steep at the red end and the other at the violet end may result in the curves intersecting between C and F, so that achromatism with those glasses would be secured for a third point in the spectrum in addition to C and F (see "Jena Glass" pp. 33, 34, and discussion). The probability of achromatism for three points being

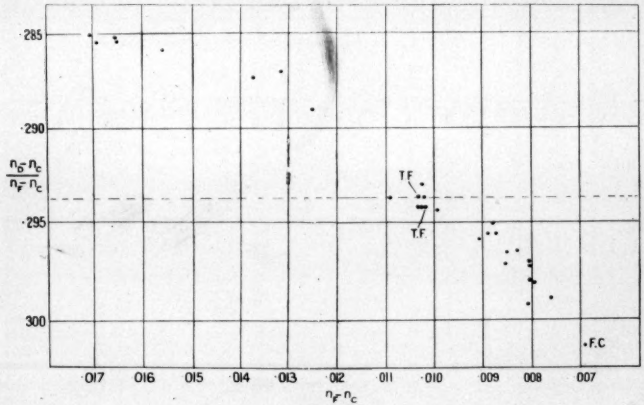


Fig. 5.

realised would be greater if the two extreme points were further apart than C and F.

It is, of course, still more desirable that the curves should match as closely as possible, as has been mentioned is the case with 17 and 12 or 18, but if possible with a greater difference in dispersive power.

In an appendix by one of us (I.M.M.) the nature of the curve of relative dispersion differences is discussed as deduced from the electromagnetic theory.

The curves of differences of relative dispersion which show the extent of secondary spectrum in a doublet also indicate clearly the method adopted by Mr. Dennis Taylor to obtain more perfect achromatism by

the use of three lenses. The triple objective as developed by him consists essentially of a convex crown and a *convex* flint combined with a concave intermediate glass. The convergent part of the system therefore gives a run of dispersion made up of the compounded dispersions of the crown and flint lenses which together match very closely the dispersion of a glass of intermediate dispersive power which is chosen of suitable index to give the system the required convergence.

If the curves of difference of relative dispersion were exactly similar in shape for all the glasses, *i.e.*, if the departures from the standard relative dispersions were always in the same proportion, then any two extreme glasses should combine theoretically with a suitable intermediate glass to give perfect achromatism (neglecting thickness). The curves however are in many cases not strictly proportional, though much similarity exists, particularly if we omit certain borate glasses of low dispersion, as might indeed be expected from the fact that the relative dispersions of three actual glasses are equal to those of the mean values of these quantities. The number of suitable glasses therefore becomes rather more restricted, and it becomes desirable to select two glasses whose dispersions compounded in some particular ratio match that of a third glass.

In selecting suitable glasses for constructing triple objectives the method here put forward may prove of value in conjunction with the mean relative dispersions as a basis for smoothing out experimental errors from a set of observations and for exhibiting in the plainest way the types of dispersions of the glasses under consideration.

In conclusion it may be said that to a certain extent the results of the investigation are disappointing in that the simplest possible relations are not found to hold good with accuracy. Previous conclusions have not been widely modified as a result of the examination of these reliable data, but the method of treatment of the facts may be suggestive and helpful in further work.

The data here considered could hardly be sufficient to draw conclusions as to the specific effect of different constituents on the dispersion of a glass, and, moreover, analyses of most of the glasses were not available and their composition could only be conjectured from the optical data and description in the Jena list.

It is hoped however that the present paper may be a prelude to a fuller investigation of glasses of known composition, possibly with attention to dispersion beyond the visible spectrum. Whilst, however, from such an investigation on the effect of various constituents facts of theoretical interest might emerge, it is improbable in view of the large amount of time hitherto devoted to the problem at Jena and elsewhere, that any great progress of practical value would result.

Our sincere thanks are due to Col. Gifford for placing so willingly in our hands the results of his tedious, lengthy and most valuable investigations, for the interest he has taken in the work, and the courtesy he has shown in consenting to re-measure the indices for certain lines in a few special instances.

APPENDIX.

Calculation of the Shape of the Curves.

On the electro-magnetic theory of light the dispersion formula connecting refractive index n and wave length λ is

$$n = \mathrm{H}\lambda^2 + \mathrm{A} + \frac{\mathrm{B}}{\lambda^2} + \frac{\mathrm{C}}{\lambda^4} - \quad - \quad - \quad - \quad - \quad - \quad (1)$$

the capitals denoting constants—where the red absorption band is so distant that only the first order term $\mathrm{H}\lambda^2$ is retained, and the violet band is rather nearer so that the second order term is retained.

If (1) is true, the relative dispersion $y = \dfrac{n - n_C}{n_F - n_C}$ must obey a law of the same type. To test this four simultaneous linear equations

$$\mathrm{K}\lambda^2 + \mathrm{P} + \frac{\mathrm{Q}}{\lambda^2} + \frac{\mathrm{R}}{\lambda^4} = y$$

were solved algebraically to find K, P, Q, R in terms of

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \text{ and } y_1, y_2, y_3, y_4$$

In the choice of the four wave-lengths two points must be borne in mind. First, as different observers obtain results measured to six figures but differing in most cases in the fifth figure, it is desirable to obtain four measurements by one observer. Secondly, as the result involves the six differences between pairs of wave-lengths, the four should be approximately equidistant. The lines chosen were B^1, D, F, and G^1 from Kaye and Laby's tables. (In completing the calculations A^1, Λ, Φ, and Cd_7 were taken from Landolt.) The coefficients then being completely determined, retaining 7 figures, it was a simple matter to test any formula of the type (1). The results were :—

$$\begin{aligned}
\mathrm{K} &= \quad 10^{0}(2{\cdot}008115y_1 - 4{\cdot}504209y_2 + 3{\cdot}995499y_3 - 1{\cdot}499404y_4) \\
\mathrm{P} &= -\ 10(1{\cdot}550242y_1 - 4{\cdot}161396y_2 + 4{\cdot}134661y_3 - 1{\cdot}623507y_4) \\
\mathrm{Q} &= \quad 10^{-8}(3{\cdot}855865y_1 - 11{\cdot}55465y_2 + 13{\cdot}29732y_3 - 5{\cdot}598544y_4) \\
\mathrm{R} &= -10^{-17}(3{\cdot}104782y_1 - 10{\cdot}01024y_2 + 13{\cdot}04803y_3 - 6{\cdot}142576y_4)
\end{aligned}$$

giving for the standard dispersions

$$y = -\ 10 \times 3{\cdot}3960\underline{\lambda^2} - {\cdot}89221 + \frac{10^{-9} + 4{\cdot}2414}{\lambda^2} + \frac{10^{-19} \times 9{\cdot}928}{\lambda^4}$$

the underlined figures being doubtful. The conclusion is that, considering the uncertainty of some of the wave lengths such as A^1, the relative dispersions do obey the given law. For, if we use these calculated dispersions to obtain a calculated refractive index n for 12, one of the three glasses having "standard" relative dispersions, and compare it with the smoothed refractive index, the difference is in the 6th decimal place. Hence the law (1) is true to the 5th decimal place (or 6th figure), *i.e.*, as far as one would expect with the available accuracy of the wave-lengths. The results are :—

	y calc$^{d.}$	y	n calc$^{d.}$	smoothed n.	n observed.
A^1	− ·34545	− ·34348	1·561454	1·561474	1·56145
B^1	(− ·172194)	− ·17219	1·563228	1·563228	1·56322
Li	− ·05334	− ·05368	1·564444	1·564441	1·56444
C	− ·00020	0	1·564988	1·564991	1·56499
D	(·293548)	·29355	1·567994	1·567994	1·56797
Λ	·45011	·44960	1·569597	1·569592	1·56959
Hg_1	·54052	·54041	1·570522	1·570521	1·57048
E	·66968	·67001	1·571844	1·571848	1·57185
F	(·999993)	1	1·575225	1·575225	1·57522
Φ	1·17877	1·17929	1·577055	1·577060	1·57706
Cd_7	1·48131	1·47923	1·580151	1·580130	1·58015
G^1	(1·574924)	1·57494	1·581109	1·581110	1·58110
Hg	2·01227	2·01583	1·585586	1·585622	1·58562

The last column of figures were those supplied by Colonel Gifford—with a note stating that ·00001 was to be added for instrumental error. A bar denotes that the value is below that stated, *e.g.*, 1·56322—lies between 1·563215 and 1·563220

It is noteworthy that the above method of deducing n from a calculated y is more accurate than calculating n directly from (1), K, Q, R being determined to 5 figures with the same accuracy that H, B, C are determined to 3 figures. This is because y varies from − 0·3 to + 2·0 while n changes only in the third significant digit. The algebraic method of solution shows the danger of asserting rashly that any particular dispersion formula does not hold, attention being paid to arithmetical values alone.

Suppose x represents the standard relative dispersions and $\eta = y - x$ the smoothed ordinate of the plotted curves. Then we have

$$x = K_1 \lambda^2 + P_1 + \frac{Q_1}{\lambda^2} + \frac{R_1}{\lambda^4}$$

$$\eta = K_2 \lambda^2 + P_2 + \frac{Q_2}{\lambda^2} + \frac{R_2}{\lambda^4}$$

Eliminating λ we obtain a general cubic relation between η and x.

If we neglect η^2 and η^3 (which are small compared with x) we obtain

$$\eta = \frac{x(x-1)(ax+b)}{px^2 + qx + r}$$

since $\eta = 0$ when $x = 0$ and when $x = 1$; a, b, p, q, r being known functions of the constants K, P, Q, R.

It has been shown that the curves are not in general very similar.

For the fluor crown 28

$$\eta = x(x-1)(\cdot 008434x - \cdot 04031)$$

so that $p = 0$ $q = 0$ $r = 1$. But the heavy flint 37 and boro-silicate crown 35 will not obey a law of the type

$$\eta = x(x-1)(ax+b).$$

This is to be expected on theoretical grounds—as the constants Q, R are relatively more important in the heavy flints due to the nearer approach of the violet absorption band—and hence these glasses would not be likely to give a similar curve on the elimination of λ.

When better values of the wave-lengths are available it will be worth while to consider further the development of these relations.

DISCUSSION.

Mr. S. D. CHALMERS said he thought Mr. Lamplough was not likely to get valuable results so long as he confined his attention to the visible spectrum. More valuable deductions could be drawn from the results of measurements in the ultra-violet and infra-red, although such measurements would be somewhat difficult to make.

Prof. A. E. CONRADY said that the author's treatment of the subject, as indicated in the diagrams, was almost exactly the same as he himself had put before his students recently. He had drawn very much the same conclusions. He did not think, however, that perfect achromatism in a triple objective depended on the conditions suggested by the paper.

Mr. T. SMITH said he had drawn similar diagrams some years ago, making use of Col. Gifford's results. He thought that the observed points departed more from the mean curves than in the examples shown by the author, and he was inclined to think that Col. Gifford had considerably overestimated the reliability of the results. There was one relation concerning the partial dispersions which he thought was worth noting, viz. :—

$$(A^1 - D) + (F - G^1) = 1{\cdot}212\,(C - F)$$

The factor was constant to within 2 or 3 in the 3rd decimal place for all the glasses in the Jena list, except in the case of one specimen, where the departure was rather greater. The result of this relation was that, if the values of n_D and ν were known, the refractive indices for the other lines could be immediately deduced.

(Note added subsequently.)—The departures from the above relation in the figures given in this paper are greater than in the figures of the Jena list from which the relation was derived.

Lt.-Col. J. W. GIFFORD communicated the following remarks :—

Any system of interpolation dependent on the use of the mean dispersion ($\mu_r - \mu_c$ or its equivalent) is greatly invalidated by there being only 3 or 4 significant figures remaining. For instance, let $\mu_r = 1{\cdot}51794$ $\mu_c = 1{\cdot}50990$, then $1{\cdot}51794 - 1{\cdot}50990 = 0{\cdot}00804$. Again, let $\mu_r = 1{\cdot}63111$ $\mu_c = 1{\cdot}61413$, then $1{\cdot}63111 - 1{\cdot}61413 = 0{\cdot}01698$. This means that the interpolation, on which the checking or smoothing described by Mr. Lamplough depends, is only reliable as far as the 3rd or 4th figure respectively, and any curves drawn, were the indices so far correct, would naturally be smooth curves.

Provided the refractive indices go far enough, no doubt Mr. Lamplough's method will be of use. A simpler formula of the same class has been used by Mrs. Gifford for checking my own measurements for some years. Measurements are first made of the refractive indices of the unknown glass for the lines limiting the mean dispersion, in our case C and F. Then a measured glass whose mean dispersion is the nearest to this is chosen from the table and the differences as given in the accompanying table found. Then, by a simple proportion between the first mean dispersion and the second, corresponding differences for the measured glass are established and from them the new refractive indices deduced. Two such cases which had already been measured were taken and the differences between measured and interpolated indices found. These are shown in the accompanying table.

We have never found either Cauchy's or Ketteler's formula of much use. By far the best of this kind of formula are Conrady's and Nelson's, but they take a long time to work out. Those referred to earlier take

far less time; for those in the table the whole operation was complete in three-quarters of an hour.

TABLE.

Formula.		*Errors.*	
$F-C : F^1-C^1 :: D-C : D^1-C^1$		Crown.	Flint.
Differences used			
$C-A^1$ $C-B^1$ $C-Li$		+ ·000031 A^1	− ·000007
$D-C$ $A-C$		+ ·000037 B^1	+ ·000003
$F-Hg_1$ $F-E$		+ ·000033 Li	+ ·000010
Cd_6-F Cd_7-F_1 G^1-F		+ ·000002 D	+ ·000004
Hg_2-F		− ·000002 A	− ·000014
All differences begin with		− ·000003 Hg_1	+ ·000008
·00 or ·01		− ·000031 E	+ ·000017
Known F − C	Unknown F − C	− ·000045 Cd_6	− ·000020
Crown.	Crown.	− ·000047 Cd_7	− ·000066
·008068	·008019	− ·000049 G	− ·000149
Flint.	Flint.	− ·000054 Hg_2	− ·000123
·016974	·017093	F − C = ·008019	F − C = ·017093

F − C and D − C refer to known glass, F^1-C^1 and D^1-C^1 to new glass, but of course F − C is known in both glasses. D − C was only taken as an example, and any other difference may be substituted for it. I think the real value of this interpolation is that there are so many lines observed that the spectrum is cut into very short lengths and the error consequently reduced, and also that there are now so many more glass observations that one with F − C like the new one can certainly be found. These glasses for interpolation were chosen haphazard.

(Additional Note.)—Referring to Mr. T. Smith's remarks, I do not quite know what statement of reliability on my part he refers to. Is he thinking about crystals which have a different order of accuracy? But may I add that Mr. Lamplough used a revised list of my glass indices, to which, thanks to some trustworthy barometer charts supplied by my friends, besides the correction for temperature, a correction for pressure had also been added. I do not think that Mr. Smith can have had access to this list.

Mr. P. F. EVERITT said he understood that the author did not hold out much hope of the possibility of constructing triple objectives corrected for three colours. Such objectives were, however, well-known commercial articles.

The PRESIDENT (Prof. CHESHIRE) said he hoped a number of such papers would be submitted to the Society. He would like to ask the author if he had found that the results for the borate flint and phosphate crown doublet in Hovestadt's book on "Jena Glass" were wrong.

Mr. LAMPLOUGH, in reply to Mr. Chalmers, said that undoubtedly investigations in the infra-red and ultra-violet would bring out more prominently variations in the relative dispersions, but measurements for these regions were comparatively rare, and the best infra-red determinations (by Rubens) did not meet properly the ultra-violet measurements of Simon for the same glass. In any case, the smoothing process described would detect errors of one in the 5th place, and as achromatism was chiefly required for the visible spectrum this seemed all that was required.

With regard to Prof. Conrady's remark that at first sight it seemed that dissimilarity rather than similarity in the curves was desirable in glasses for triple objectives, the author said he did not like to express his disagreement with even a hasty conclusion of an eminent authority, but it was however a fact that the curves referred to in the paper were not widely different for many glasses.

In reply to Mr. Smith he agreed that the accuracy of Col. Gifford's results varied somewhat. Some of the best work was that on the reference prisms for which very smooth curves were obtained. The results had a special value in that they were all absolute and the smoothing process gave an indication of the degree of accuracy of any set of measurements. With regard to Col. Gifford's contribution the authors would point out that the indices utilised give values for the mean dispersion to one in 1600 to 3400. Any limitation in the number of significant figures of the quantities used in the smoothing process would make the curves less and not more smooth. Mrs. Gifford's method of "checking" has the great objection that it assumes *ab initio* that the relative dispersions of glasses are identical when their mean dispersions are equal. As pointed out, this principle would make it unnecessary to measure more than two lines for any glass, when accurate results were available for a large number of glasses of different dispersions. Apart from this however it is not clear how the accuracy of the standard glasses for comparison can be checked. The method, though not strictly legitimate, is undoubtedly a useful check for approximate results in many cases, but will not give all that can be deduced from the method described by the authors in which all the lines contribute to the smooth curve by which any particular dispersion is corrected.

With reference to Mr. Everitt's remarks, he did not mean to convey the impression that *triple* objectives corrected for three colours could not be constructed. The curve he had referred to was for a doublet and had been taken from Hovestadt's book.

In reply to the President, the author said he thought the mistake had probably arisen from the approximate formula employed in calculating the various focal lengths.

[On looking more closely into this since the reading of the paper, it has however been found that some of the relative dispersions for the two glasses in question given in the "Jena Glass" are not correct to the nearest third significant figure, and that calculations from the indices themselves do in fact indicate equality of focal length for three points in the spectrum. (This possibility is referred to in the paper as published.) It may however be remarked that an error of only ·00001 in the determination for A, C, or D of one of the glasses would suffice to invalidate this conclusion.]

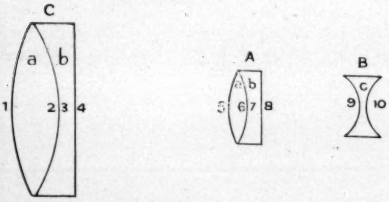
An Improved Binocular of the Galilean Type.

By HERBERT S. RYLAND.

Read 10th January, 1918.

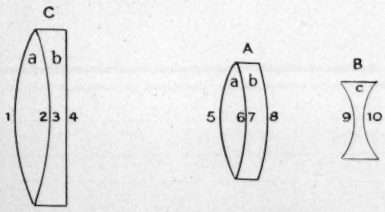
THE Galilean type of binocular has several merits and one demerit, viz., lack of field. Also it cannot readily be fitted with cross wires. In its favour are, firstly, good light transmission, also the emergent pencil is large enough to fill the pupil of the eye even when fully expanded.

Fig. 1. Small Model.



Glasses	n_D	ν
a Hard Crown ..	1·5174	58
b Dense Flint..	1·6166	37
c Light Flint....	1·5704	43

Fig. 2. Large Model.



Radii of Curvature.

Small Model.	Large Model.
1. + 50	1. + 83
2. — 53	2. — 95
3. — 53	3. — 95
4. ∞	4. ∞
5. + 32	5. + 37·6
6. — 32	6. — 37·6
7. — 32	7. — 37·6
8. —195	8. —200
9. — 34	9. — 34
10. + 15	10. + 15

	Diameters.				Separations.	
Small Model ..	C 38	A 15	B 12	..	C to A 75	A to B 20
Large Model ..	C 52	A 18	B 13	..	C to A 100	A to B 23

Secondly, it is readily taken to pieces in the field, cleaned and reassembled without spoiling the adjustment. It is, further, far less likely to get out of adjustment in service than prism glasses. Lastly, it can be manufactured more cheaply.

The object of the present design is to increase the field of view. Referring to the drawings, Fig. 1 is a section of one side of a binocular having a power of 3 and a field of 12°. Fig. 2 shows the same details for a power of 6.5 and a field of 5° 30′. Both binoculars have been made, and give good definition to the edge of the field.

It will be noted from the drawings that for a given power and field this type of binocular is smaller than usual. In Fig. 1, B is the usual eye lens and A an objective. A and B used alone, with proper separation, will give substantially the same magnification as the complete instrument. The focal length of C is chosen so that when adjusted for infinity it is at the focal point of A, while the distance from A to B is substantially the focal length of B. It is further to be noted that if the distance from A to B is made adjustable, a variable power binocular is available.

The larger of the two instruments which I have had made has this adjustment, and the power varies from 3·5 to 6·5. The radii given are correct, but will probably need adjustment as the glasses available in this country will be likely to differ somewhat from the American glasses employed in the models. Binoculars are now being made, and as soon as possible I will send one to the President for the use of the Society. All dimensions are in mms.

A Spherometer of Precision.

By J. GUILD, A.R.C.S., D.I.C., F.R.A.S. (From the National Physical Laboratory).

Read and discussed 10th January, 1918.

THE type of spherometer herein described was designed with a view to obtaining greater accuracy in the determination of the exact point of contact of the micrometer leg with the surface of the lens or flat under examination than is usually obtained with spherometers of existing patterns. The essential features of the instrument are indicated in Fig. 1.

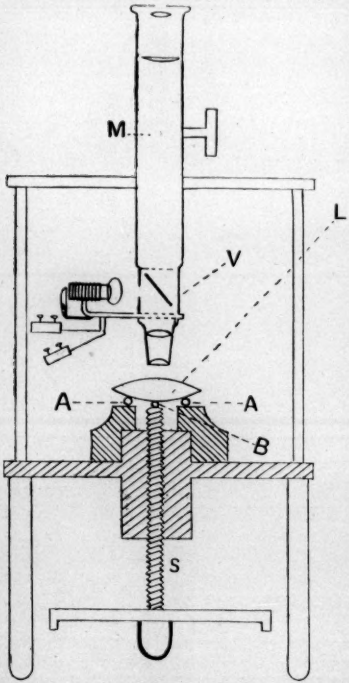


Fig. 1.

The lens or flat, L, under test rests upon three spheres, of which only two are shown at A A. These spheres are mounted upon an adapter,* so that by using different adapters the instrument is suitable for surfaces of different sizes. The micrometer S is screwed up from below: it terminates in a small spherical surface B of one to two millimetres diameter, of which the apex is exactly concentric with the screw.

When B is in contact with the surface of L a system of Newton's rings will be visible if suitable illumination and observing apparatus are provided. For this purpose there is mounted above the spherometer a microscope M, with an ordinary vertical illuminator V fitted above the objective. Illumination is supplied by a small 4-volt lamp on a bracket fitted between the

* The type of adapter in which the surface rests on a ring instead of three spheres would do equally well.

objective and vertical illuminator. A piece of Wratten No. 25 gelatine filter (red) is mounted behind the aperture of the illuminator and renders the light sufficiently monochromatic for a considerable number of interference fringes to be seen surrounding the point of contact. Filters of other colours were tried and proved equally satisfactory, though the author prefers the red field for comfort in working.

The method of using the instrument can now be briefly explained. Suppose that the small sphere B is gradually approaching the surface of L. When it comes within a short distance of it the fringes become visible on suitably focussing the microscope. As the separation diminishes the fringes expand outwards, a fresh one appearing at the centre for each half wave length that the screw advances ; but at the moment at which contact is made and the lens L begins to lift the expansion of the fringes ceases abruptly. This point can be determined with certainty to less than one five-thousandth of a millimetre. It is better, however, to use as the criterion of contact not the point at which motion of the ring system ceases but some easily recognised configuration just before this stage is reached. A convenient configuration is that in which the central black spot approximately trisects the diameter of the first black ring. This has the advantage that the appearance continues to alter for a little beyond the required setting, which is more satisfactory in practice than when the criterion adopted is one-sided. But the most important advantage in the use of this setting is in the elimination of a source of error which is inherent in any spherometer in which, when the criterion of " contact " is reached, the weight of the lens is borne either wholly or in part by the central point. In working with this instrument it is found that when the lift-point is reached and the fringes cease to expand the central spot is spread out to an extent depending on the weight of the lens. Thus with a light eye-piece lens its diameter may not exceed a third of that of the first ring ; whereas with a fairly heavy lens or flat it may almost fill the first ring. This spreading out of the black spot indicates a compression partly of the small sphere B and partly of the lens surface ; and the result is that the lift-point is not attained until the screw has advanced beyond the point of geometrical contact. Clearly, unless the lens and the flat on which the zero reading is made are of the same order of weight, an error will arise from this cause. Since the same flat has usually to serve whatever size of lens is being measured up, this condition cannot generally be fulfilled. As regards the magnitude of the error, the weight correction for a circular flat of two inches diameter and one-quarter inch thick (weight = 35 gm. approx.) was about four ten-thousandths of a millimetre with the smallest adapter. It is greater in the case of larger adapters, though this is partly compensated by the smaller difference in weight between the flat and the lenses tested on these. Errors of this magnitude are several times larger than the experimental error with this type of instrument. It is obvious

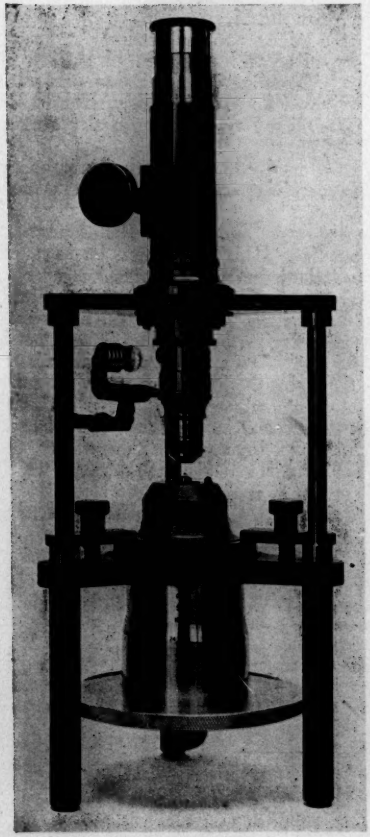


Fig. 2.

that such errors will be of much larger magnitude in instruments in which the micrometer terminates in a point instead of a sphere, so that pointed ends should be avoided in all spherometers. They have no advantages over rounded ends, and have several disadvantages in addition to the one discussed here.

With the author's instrument not only can the weight error be investigated quantitatively, but in ordinary use it can be entirely overcome by using the criterion of trisection of the first ring by the central spot; for at this stage in the approach compression has not yet commenced, and an exactly similar degree of "contact" is obtained whatever the weight of the lens or flat.

Fig. 2 is a photograph of an actual instrument adapted from an existing pattern of spherometer. The lower portion is simply the lower portion of this instrument with the addition of a milled disc of larger diameter than the drum to increase the accuracy of setting, and with the central steel point replaced by a small sphere. This sphere is of glass, because it was easier to make with the facilities at hand; but a quartz sphere would be preferable on account of its lesser liability to accidental scratches.* The

* A steel sphere was tried in the first instance; but the fringes given by this were not nearly so distinct as with a transparent material. Further, on account of the comparative roughness of the surface, the fringes were so irregular that the trisection method of setting could not be employed.

super-structure of the original instrument is replaced by that shown in the photograph, which carries the microscope and vertical illuminator. A one-inch objective and a $\times 12$ eye-piece, with a tube length of 12 to 14 cm., give a suitable magnification. The objective should have as long a working distance as possible to allow of its use with fairly thick lenses. The vertical illuminator is of the ordinary type, having a single cover glass reflector (unsilvered).

It is best to use three dry cells of moderate size for the lamp, as the small flash-lamp batteries are always giving out at the least convenient times.

With this instrument settings can be repeated to about one ten-thousandth of a millimetre with very little practice. This is of course far in excess of the accuracy of which the screw and other mechanical parts are capable; so that the measurements are by no means reliable to a degree corresponding to the sensitivity of setting. Experience with the actual instrument shows, however, that the interference ring method of detecting contact is sufficiently sensitive to obtain the utmost accuracy from an instrument of the highest mechanical perfection. Further, the accuracy is the same whatever size or type of lens is being measured—a most important point in an instrument which has to be employed for small eye-piece lenses, etc. With many, if not most of the existing types of spherometer, the accuracy with small diameter lenses is much less than with those for which the larger adapters can be used, especially if the curves are steep.

There is also a great saving of time with this instrument, one setting only requiring a few seconds.

It is of interest to investigate the possible precision of an instrument of this design. A change of a complete fringe corresponds to half a wave length of light in screw travel, *i.e.* to one four-thousandth of a millimetre approximately. Since settings can be made without difficulty to a fifth of a fringe, if not less, an accuracy of a twenty-thousandth (1/20,000) of a millimetre is attainable with ease as far as the method of setting is concerned. This, unfortunately, is far in excess of the accuracy with which micrometer screws can be manufactured or tested at present.

APPENDIX.

THE best method of making, mounting and centering the transparent sphere on the end of the micrometer screw is, of course, a matter for the instrument maker; but the method employed by the author in adapting his own instrument may be of interest to anyone who might desire to adapt an existing spherometer of suitable type for use by the interference method.

The end of the screw was turned down to a depth of about 0·5 mm. for a length of about 5 mm., Fig. 3 c, and a steel sleeve, Fig. 3 b, of 1 cm. length was made to fit firmly over the narrow portion. In the closed end of this sleeve a slightly countersunk hole of about 1 mm. diameter was bored. This was done while the sleeve was in position on the end of the

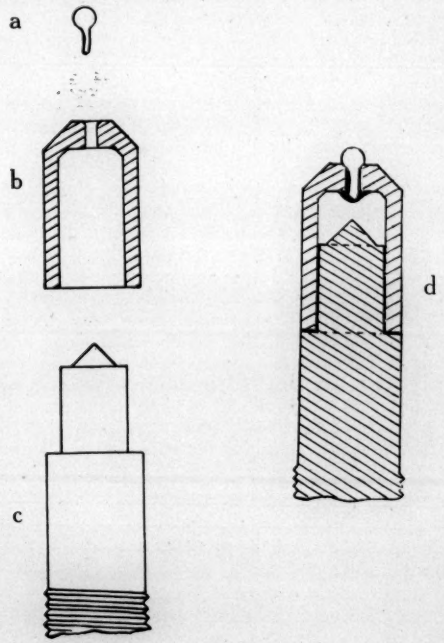


Fig. 3.

micrometer (which was fitted in a lathe) so that the hole should be exactly concentric with the screw.

The glass sphere was made by drawing out glass rod to about 0·5 mm. diameter or less, and fusing the end in the blowpipe flame until the sphere which is thus formed had grown to the required size. A number of such spheres can be turned out in a few minutes, and it is easy to find one that

is reasonably spherical, of the right size, and free from air bubbles and other defects. A sphere having been chosen, it was broken off with about an eighth of an inch of the thin rod left as a neck, Fig. 3 a.

The micrometer screw was mounted vertically in guides, one of which was close up to the sleeve, so that it could be rotated about its axis without shake. The lip of the hole was smeared with seccotine (which takes some little time to harden) and the sphere was placed in position as shown in Fig. 3 d. A strip of microscope cover glass was then supported so that it rested horizontally on the sphere and on two other supports attached to the upper guide. Thus mounted the cover glass is approximately perpendicular to the axis of the screw. A microscope with a scale in the eyepiece was mounted so as to observe the Newton's rings between the sphere and the cover glass. Any slight want of centering of the highest point of the sphere was now made evident by the movement of the rings in the field of view when the screw was rotated in the guides. This was corrected by pushing the sphere in the proper direction with a suitable implement until, on rotating the screw, no appreciable movement of the interference rings took place. The adjustment had to be repeated from time to time as the seccotine hardened, on account of uneven shrinkage. As little of the adhesive as possible should be employed. When the seccotine was perfectly hard the sleeve was removed and a mixture of beeswax and resin was melted into the upper end, filling up the small hole and completely surrounding the neck of the sphere, which was thus held in position with perfect rigidity. The sleeve was replaced on the end of the screw, and the latter returned to its place in the spherometer.

Subsequent checks of the centering adjustment have not revealed any appreciable shift due to creeping of the materials used in mounting the sphere.

Note added January 10th, 1918.—It has been suggested that details should be given of the method of centering the adapters and of measuring the radius of the circle on which the three balls lie. The centering is done once and for all by the makers, the adapters being such an accurate fit that they always return exactly to the same position.

There are two convenient methods of measuring the radius of the circle on which the centres of the three spheres lie. The first of these is to measure, with a Brown and Sharpe micrometer gauge, the distances l, m and n [Fig. 4 (a)]. Subtracting the diameter of the balls from each of these gives us the three sides of the triangle at the corners of which the centres of the balls lie, and the radius of the circumcircle is calculated from these. As a check on this value the distances u, v and w [Fig. 4 (b)] are also measured. By subtracting from the mean of these the radius of a ball and also the radius of the cylindrical end of the spherometer screw (the measurement, of course, being made near the end where there is

no thread) the radius of the circumcircle is again obtained. This measurement should not be made from the sides of the central sphere, as this is unlikely to be sufficiently spherical to give accurate values.

The second method of measuring the distances between the various spheres is to lay an optical flat on them and measure with a travelling microscope the distances between the centres of the Newton's ring systems formed at the contacts.

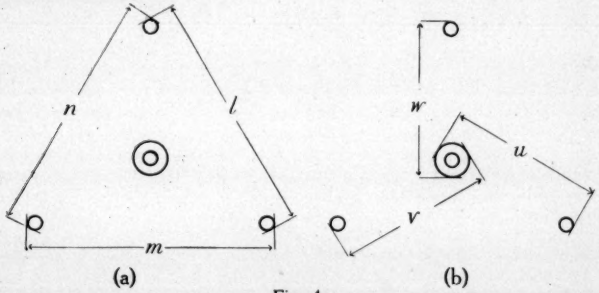


Fig. 4.

If R is the radius of the circle through the centre of the three balls, and h is the displacement from its zero position of the micrometer point when making contact with a surface of radius r resting on the three balls, $r = R^2/2h + h/2 \pm a$, where a is the radius of the balls. The + sign applies to concave, and the — sign to convex surfaces.

For a ring adapter the diameter of the two significant edges of the ring can be measured on a travelling microscope.

DISCUSSION.

MR. P. F. EVERITT said that the author had only referred to a spherometer in which the micrometer screw was rotated until the specimen under test ceased to "wobble." The only kind of instrument of this type that he knew was a toy. He himself had used two different types of reasonably accurate spherometers. The first was one made by Zeiss, consisting of a ring and a central plunger with a counterweight. A scale is engraved on the plunger and is observed with a microscope provided with a micrometer eye-piece. The other spherometer was a Steinheil

instrument. It is of the three-sphere type and has a central plunger with a sphere at each end. The plunger is driven up by a lever on a slide which is actuated by a measuring screw. The lever is provided with a counterweight. In both these instruments constant pressure can easily be obtained. He thought that Mr. Guild would get as great accuracy if he applied his method to constant pressure spherometers. He considered that the author would require to make further corrections for the pressure on the three spheres.

Mr. S. D. Chalmers said he had had experience with the Abbe spherometer. The weak point in it is the ring and not the central point. It is difficult to keep the diameter of the ring constant, and since the edge must be sharp it bites into the lens. He thought Mr. Guild deserved great credit for his ingenious method of increasing the accuracy of measurement of curvature, especially in the case of small lenses. He would like to call the author's attention to the fact that the principle involved in his method had been applied to a different instrument, viz., an extensometer, which had been described in the Proceedings of the First Optical Convention.

Prof. A. E. Conrady drew attention to the denting by the three spheres. If the central ball could be of the same size and compressibility as the other three, the pressures at each sphere could be calculated from the measurements obtained when contact was made and when the lens was just lifted up by the central sphere.

Mr. Guild, replying to Mr. Everitt, said he did not mean to infer that the lifting type of spherometer was the only one used. He had had no experience with the Zeiss or Steinheil instruments, but thought that, under present circumstances, there were obvious advantages in having an accurate instrument of British design. He had seen the Zeiss spherometer, but, from his experience of micrometer eye-pieces, he did not think that as great accuracy could be obtained with it as with the instrument described.

The question of the pressure at the three spheres was an obvious one. He had investigated the point by making measurements with an optical flat, first by itself, and then with metal rings of comparable weight placed on top. The results showed that the error due to compression at the three spheres was very much less than in the case of the central sphere. He had not found that any correction comparable with the error of observation with his actual instrument was necessary.

He was unaware that the principle involved had been applied to an allied type of instrument.

In reply to the President he stated that all observers could get the same readings to within 1/5000 mm. without previous experience. The instru-

ment to which he had applied his method was one of the toys referred to by Mr. Everitt; the screw was not a good one. Nevertheless, with practice an observer repeated his readings to about 1/10000 mm.

MR. CHALMERS remarked that the accuracy of the Abbe instrument was about 1/2000 mm.

MR. EVERITT said the other type he had mentioned gave about the same degree of accuracy.

THE PRESIDENT (Prof. Cheshire) said that the accurate measurement of deep curves was of great importance to optical workers.

ABSTRACT.

"CHALDEA." Zenaide A. Ragozin. (Published by T. Fisher Unwin, Paternoster Square, 1891.)

In this book, which is treated as a general introduction to the study of Ancient History, it is assumed that glass lenses were used for magnifying purposes in 4000 B.C.

Layard, in his first description of his startling "find," says, "They (the tablets) were of different sizes; the largest were flat, and measured nine inches by six and a half; the smaller were slightly convex, and some were not more than an inch long, with but one or two lines of writing. The cuneiform characters on most of them were singularly sharp and well defined, but so minute in some instances as to be illegible without a magnifying glass." Most curiously, glass lenses have been found among the ruins, which may have been used for the purpose. Specimens have also been found of the very instruments which were employed to trace the cuneiform characters. (Page 109.)

They (the small cylinders) are sometimes shaped like a prism with several faces (mostly six), sometimes like a barrel, and covered with that compact and minute writing which it often requires a magnifying glass to make out. (Page 114.)

"Chaldea" points to a monumentally recorded date nearly 4000 B.C. (Page 364.)

The book mentions a large collection of these cylinders at the British Museum, but no mention is made whether the lenses found were preserved. J. P.

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